PROGRAM

Thursday 3 November 2016

8h30-9h00 Registration

9h00-9h30 Welcoming address by the organizers

9h30h-11h00 Jeremy Avigad (Carnegie Mellon University), *Mathematical Understanding*

11h00-11h30 Pause

11h30-13h00 Gabriel Tarziu (Institute for Research in Humanities, Universitatea din Bucuresti), *Can we have mathematical understanding of physical phenomena?*

13h00-14h30 Lunch

14h30-15h15 David Waszek (Université Paris I Panthéon-Sorbonne, Institut d'Histoire et de Philosophie des Sciences et des Techniques), *On the autonomy of formulae: a case study around Leibniz and Johann Bernoulli*

15h15-16h45 Sébastien Maronne (Univ. Toulouse III Paul Sabatier, IMT & SPHERE), *Philosophy of mathematics between history of mathematics and philosophy. Geometrical problem solving in early modern mathematics and practical reasoning*

16h45-17h15 Pause

17h15-18h45 Francesca Boccuni (Università San Raffaele, Milan) – Jack Woods (University of Leeds), *What Should we Render unto Caesar?*

Friday 4 November 2016

9h30h-11h00 Christophe Eckes (Archives Henri Poincaré, Nancy), *Albert Lautman lecteur de Hermann Weyl*

11h00-11h30 Pause

11h30-13h00 Francesca Biagioli (Universität Konstanz), *Arithmetization as a tool of discovery in Felix Klein's research program and epistemological writings*

13h00-14h30 Lunch

14h30-15h15 Antonio Piccolomini d'Aragona (Aix-Marseille University, University of Rome "La Sapienza"), *A case of BHK decidability: Dag Prawitz's proof- and ground-theoretic semantics*

15h15-16h45 Claudio Bartocci (Università di Genova), *From Euclid to* F_1-*geometry*

20h30 Social Dinner

Saturday 5 November 2016

9h30h-11h00 Jean-Michel Salanskis (Université Paris Ouest), *Category Theory and Philosophy of Mathematics*

11h00-11h30 Pause

11h30-13h00 Simon Hewitt (Department of Philosophy, University of Leeds), *Linguistic Reinterpretation and Indefinite Extensibility*

BOOK OF ABSTRACTS

Jeremy Avigad (Department of Philosophy and Department of Mathematical Sciences, Carnegie Mellon University

Mathematical Understanding

In the analytic tradition, the philosophy of mathematics has generally focused on justification, exploring the grounds for mathematical knowledge and the proper methods of inference. From that perspective, the body of mathematical knowledge consists of something like the collection of definitions, theorems, and proofs that have been accumulated over the years. It is not difficult to make the case that there are important mathematical resources that cannot be characterized in these terms, but merit philosophical attention nonetheless. A theory of mathematical *understanding*, in contrast to a theory of mathematical knowledge, should provide a general account of these epistemological resources. In this talk, I will clarify some of the motivating intuitions and goals, and offer some suggestions as to how we ought to proceed.

Claudio Bartocci (Università di Genova)

From Euclid to F1-geometry

From ancient Greece to the present, the elusive concept of "point" has been extensively debated throughout the entire history of mathematics: just to cite a few obvious examples we may consider the problem of the continuum, the interplay between local and global structures in differential geometry, the notion of "point" in algebraic geometry or topos theory. In which sense, if any, spaces are made up of points?

Francesca Biagioli (Universität Konstanz)

Arithmetization as a tool of discovery in Felix Klein's research program and epistemological writings

It is well known that Felix Klein articulated one of the first unified approaches to geometry in group-theoretical terms in 1872. However, less attention has been given to the development of Klein's ideas from his projective model of non-Euclidean geometry of 1871 to his renewed interest in this topic in the 1890s. This paper reconsiders the philosophical motivations of Klein's research program in the light of his later considerations about the so-called arithmetization of mathematics. Klein deemed this the program of a rigorous foundation along the lines of Dedekind's definition of irrational numbers as singular positions in arithmetical structures. It followed that, contrary to the received view, the existence of irrationals does not depend on our intuitions concerning the continuity of the line. Even though Klein believed that intuitions play an important role in mathematics teaching and in mathematical practice, he defended Dedekind's approach to foundational issues. Klein used his concept of a projective metric as an example of how the same methodology can be used in geometry. My suggestion is that Klein attached a special importance to the numerical representation of projective space, because, as he made clear in a series of writings from the 1890s, this opened the door to a general classification of metrical geometries. Klein believed, furthermore, that the same classification shed light on the different hypotheses concerning physical space.

The first part of the paper provides basic information about Klein's projective model of non-Euclidean geometry and uses this example to discuss his epistemological ideas. The second part deals with the philosophical reception of these ideas by Bertrand Russell and Ernst Cassirer. I rely on Cassirer to emphasize some of the aspects overlooked by Russell and articulate the suggestion that arithmetization played the role of a tool of discovery in the development of Klein's thought.

Francesca Boccuni (Università San Raffaele, Milan) – **Jack Woods** (University of Leeds)

What Shoud we Render unto Caesar?

We will tackle the so-called Julius Caesar problem, which concerns the inability of Hume's Principle (HP), Frege's way of carving out the cardinal numbers, to distinguish between objects of different sorts. HP says that the number of Fs is the same as the number of Gs if and only if F and G are equinumerous – i.e. can be put into 1-1 mapping. In Frege's provocative example, how can we know that the reference of "the number of the buildings in Saint Charles campus in Marseille", as specified by HP, picks out the cardinal number *n* instead of Julius Caesar? We will suggest a solution to this vexed problem in terms of arbitrary reference. We go on to show that under certain assumptions, abstraction operators such as those generated by HP are *isomorphism-invariant*, underwriting the attribution to them of a privileged logical status.

Christophe Eckes (Archives Henri Poincaré, Nancy)

Albert Lautman lecteur de Hermann Weyl

Dans cet exposé, nous entendons tout d'abord offrir une vue d'ensemble des sources mathématiques convoquées par Albert Lautman en vue de la constitution de sa philosophie mathématique. Pour ce faire, nous insisterons sur ses liens avec les mathématiciens Jacques Herbrand – qui décède en 1931 — et Claude Chevalley. Nous verrons en outre que Lautman accorde un intérêt tout particulier aux deux grandes monographies qui paraissent en topologie : le *Lehrbuch der Topologie* de Herbert Seifert et William Threlfall (1934) ainsi que la Topologie de Pavel Alexandrov et Heinz Hopf (1935). Nous montrerons dans quelle mesure Lautman se réfère à *Die Idee der Riemannschen Fläche* de Weyl àtravers le prisme de ces monographies très récentes en topologie. Pour finir, nous essaierons de comprendre comment Lautman interprète la préface à *Gruppentheorie und Quantenmechanik* de Weyl (première édition 1928, deuxième édition 1931). Cette interprétation, qui conditionne l'architecture d'ensemble de la thèse complémentaire de Lautman, est discutée par Paul Bernays dans sa recension des thèses de Lautman et elle est finalement portée à la connaissance de Weyl en 1939.

Simon Hewitt (Department of Philosophy, University of Leeds)

Linguistic Reinterpretation and Indefinite Extensibility

The thought that the extensions of some mathematical concepts - such as 'set' or 'ordinal' - is indefinitely extensible - is familiar from the work of Michael Dummett. One way of cashing out indefinite extensibility is in terms of an ontological open-endedness. This approach has recently been made rigorous by Linnebo.

An alternative owing to Uzquiano understands indefinite extensibility in terms of our capacity to reinterpret our language such that further objects fall within the extension of 'set'. After first reviewing the proposal, I argue that this approach is best understood as a version of in rebus structuralism and is therefore subject to familiar difficulties for that approach to the philosophy of mathematics. In particular, there is no reason independent of an antecedent acceptance of set-theory to believe that there are enough objects for a reinterpretation approach to recover ZFC2.

http://www.simonhewitt.org/

Sébastien Maronne (Univ. Toulouse III Paul Sabatier, IMT & SPHERE)

Philosophy of mathematics between history of mathematics and philosophy. Geometrical problem solving in early modern mathematics and practical reasoning.

When confronted with a geometrical problem to solve, early modern mathematicians had to build strategies in order to provide a *legitimate* construction of the solution. In his article « Philosophical Challenges from History of Mathematics », Henk Bos stresses that the legitimacy issue under discussion deals with the *procedures* which exhibit mathematical objects and not with the ontology of these objects. This leads him to bring into focus the mathematical *tasks* rather than the *concepts* which result from the performance of these tasks and to conclude by the following question: « Where, in present-day or earlier philosophy of mathematics, can I find explorations of a view of mathematics as the performance of self-imposed tasks? ». In my talk, I will try to address this question by considering jointly the part of the abundant literature on philosophy of action which studies the logic of practical reasoning and early modern mathematics sources taken from Descartes, Pascal, Leibniz and Huygens. I will focus in particular on the issues concerning instrumental (Aristotle) and normative (Kant) viewpoints on practical reasoning.

Antonio Piccolomini d'Aragona (Aix-Marseille University, University of Rome “La Sapienza”)

A case of BHK decidability: Dag Prawitz's proof- and ground-theoretic semantics

According to a widespread proposal, the meaning of the logical constants must be explained through an epistemic key-notion accounting for evidence. The BHK semantics probably constitutes the most discussed development of this idea.

Here, implications and universally quantified formulas are dealt with via effective operations that produce proofs of a specific kind when applied to arguments in their domain. Since no upper bound is put on the complexity of these operations, it is possible that being in possession of them does not also amount to seeing that they yield desired results. However, an understanding of this kind must be required, for it would be unreasonable to have proofs which we cannot recognize as such. But since the BHK provability relation is not decidable, the question arises about what such an understanding should exactly be.

Famously, Dag Prawitz has developed a constructive semantics meeting the demands of a verificationist theory of meaning. Within this context, he has discussed the BHK decidability problem (Prawitz 1977), and shown how an analogous issue lurks out when evidence is described by means of valid arguments (Prawitz 1973) or ground-terms of a suitable typed and extended λ-language with equational axioms (Prawitz 2015).

In my talk, I will mainly focus on Prawitz's semantics. In the first part, I will try to argue that, thanks to the radically new description that the theory of grounds provides of proofs and inferences, the ground-theoretic version of the decidability question is somehow less problematic.

Finally, I will try to frame the expressions “decidable” and “understanding” in appropriate ways – by ruling out problematic solutions like Kreisel's second clause and “algorithmic” proposals – and try to see whether, based on such framings, one may be in the position to speak of (possibly weaker kinds of) “decidability” and “understanding”.

Bibliography

- D. Prawitz 1973, *Towards a foundation of a general proof-theory*, in P. Suppes, *Logic methodology and philosophy of science IV*, Amsterdam, North-Holland Publishing Company, 225 - 307.
- Id. 1977, *Meaning and proofs: on the conflict between classical and intuitionistic logic*, in *Theoria* 43, 2 – 40.
- Id. 2015, *Explaining deductive inference*, in H. Wansing, *Dag Prawitz on proofs and meaning*, Heidelberg, Springer, 65 – 100.

Jean-Michel Salanskis (Université Paris Ouest)

Category Theory and Philosophy of Mathematics

Over the last decades, it has been often argued that, in a sense, category theory was radically changing everything concerning mathematics: that it had already observably transformed mathematics from the inside, that it was suggesting new foundations, that it was reinterpreting the frontier between logic and mathematics, that it was offering a third position between formalism and constructivism, to name a few claims.

The aim of this talk is simply to take stock of such issues: to define possible ways of discussing them on the one hand, and, on the other hand, to try and understand better what category theory indeed brings and what it is about.

Gabriel Tarziu (Institute for Research in Humanities, Universitatea din Bucuresti)

Can we have mathematical understanding of physical phenomena?

It is generally acknowledged that mathematics plays a remarkably fruitful role in science, but how helpful can it be when it comes to our scientific quest for understanding the physical world? Can mathematics contribute to our understanding of physical phenomena? Some philosophers (e.g. Colyvan (2001), Baker (2005, 2009, 2012), Lyon (2012)) offer what can be taken as good grounds for an affirmative answer; they argue that we can find in science examples of explanations in which the mathematical part is doing a genuinely explanatory job. But, as others argue (e.g. Melia (2002), Daly and Langford (2009), Saatsi (2011)), there are powerful reasons for not taking such examples as cases of mathematical explanations of physical phenomena. The problem with this last position is that it fails to account for the intuition that, in the examples discussed, the mathematical part contributes somehow to the understanding of the physical fact in question. We seem to be, then, in the following situation: we have many examples of explanatory answers to scientific questions that *prima facie* seem to make essential appeal to mathematics in order to convey understanding about some physical fact. Some philosophers argue unconvincingly (in my and others' opinion) that these examples should be taken as cases of mathematical explanations, others disagree and argue that mathematics plays other non-explanatory roles in these examples loosing on the way the possibility to account for the intuition that mathematics is important for understanding in these cases. My aim is to offer a way out of this situation by giving an account that takes mathematics as conveying understanding in such contexts even though it is not explanatory. My position is in an important respect similar to Mark Steiner's (1978) account. I believe that there is something that 'leaks' from the purely mathematical context to the physical one, but it is not explanatory power. What gets transmitted is, in my view, (mathematical) understanding.

David Waszek (Université Paris I Panthéon-Sorbonne, Institut d'Histoire et de Philosophie des Sciences et des Techniques)

On the autonomy of formulae: a case study around Leibniz and Johann Bernoulli

Sometimes, apparently nonsensical computations lead to mathematical discoveries. This is (partly) how, in 1695, Leibniz and Johann Bernoulli discovered the rudiments of what we now call the "calculus of operations". First, Leibniz noticed a vague analogy between higher differentials and powers. Johann Bernoulli then tried strange symbolic manipulations suggested by this analogy, and unexpectedly arrived at correct results. Later, in the eighteenth century, Lagrange, Laplance, Arbogast and others developed Leibniz and Bernoulli's methods and attempted to put them on a sound footing.

In the end, one can make good sense of Johann Bernoulli's apparently absurd computations. But this requires re-interpreting his formulae and introducing new objects: the integral and differential operators. Under the meaning Leibniz and Bernoulli themselves gave to their symbols, Bernoulli's manipulations are incomprehensible.

We will discuss this episode and explore its consequences for our understanding of symbolic formalisms.